

FIG. 1

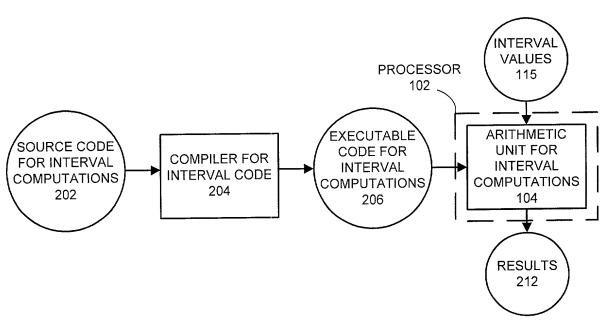


FIG. 2

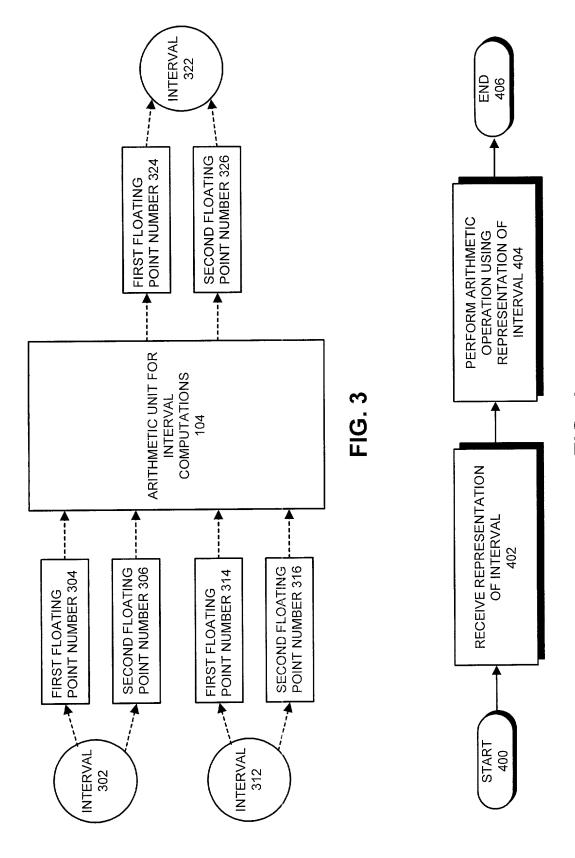


FIG. 4

$$X = \left[\underline{x}, \overline{x}\right] = \left\{x \in \Re^* | \underline{x} \le x \le \overline{x}\right\}$$

$$Y = \left[ \underline{y}, \overline{y} \right] = \left\{ y \in \Re^* | \underline{y} \le y \le \overline{y} \right\}$$

(1) 
$$X+Y = \left[\sqrt{\underline{x}} + \underline{y}, \uparrow \overline{x} + \overline{y}\right]$$

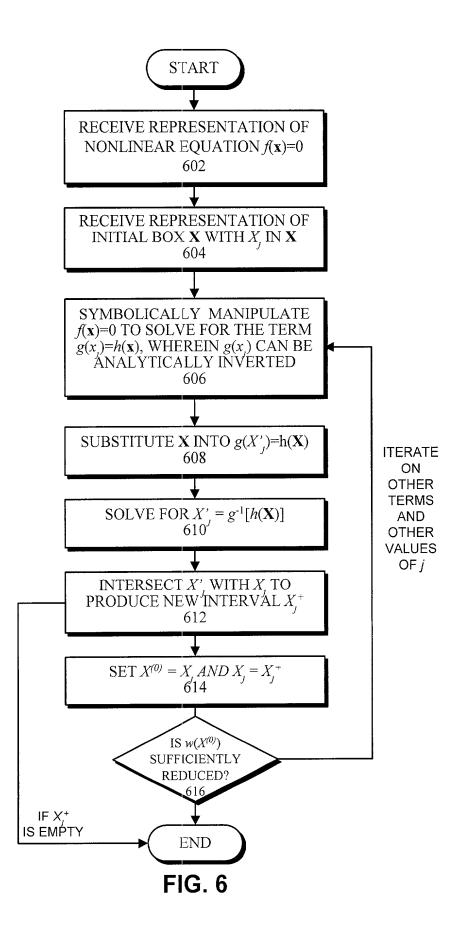
(2) 
$$X-Y = \left[ \sqrt{\underline{x}} - \overline{y}, \uparrow \overline{x} - \underline{y} \right]$$

(3) 
$$X \times Y = \left[ \min\left( \sqrt{\underline{x}} \times \underline{y}, \underline{x} \times \underline{y}, \overline{x} \times \underline{y}, \overline{x} \times \underline{y} \right), \max\left( \uparrow \underline{x} \times \underline{y}, \underline{x} \times \underline{y}, \overline{x} \times \underline{y}, \overline{x} \times \underline{y} \right) \right]$$

(4) X/Y=
$$\left[\min\left(\sqrt{\underline{x}}/\underline{y}, \overline{x}/\underline{y}, \overline{x}/\underline{y}, \overline{x}/\underline{y}\right), \max\left(\sqrt{\underline{x}}/\underline{y}, \overline{x}/\underline{y}, \overline{x}/\underline{y}, \overline{x}/\underline{y}\right)\right], 0 \notin Y$$

$$X/Y \subseteq \Re^*$$
,  $0 \in Y$ 

FIG. 5



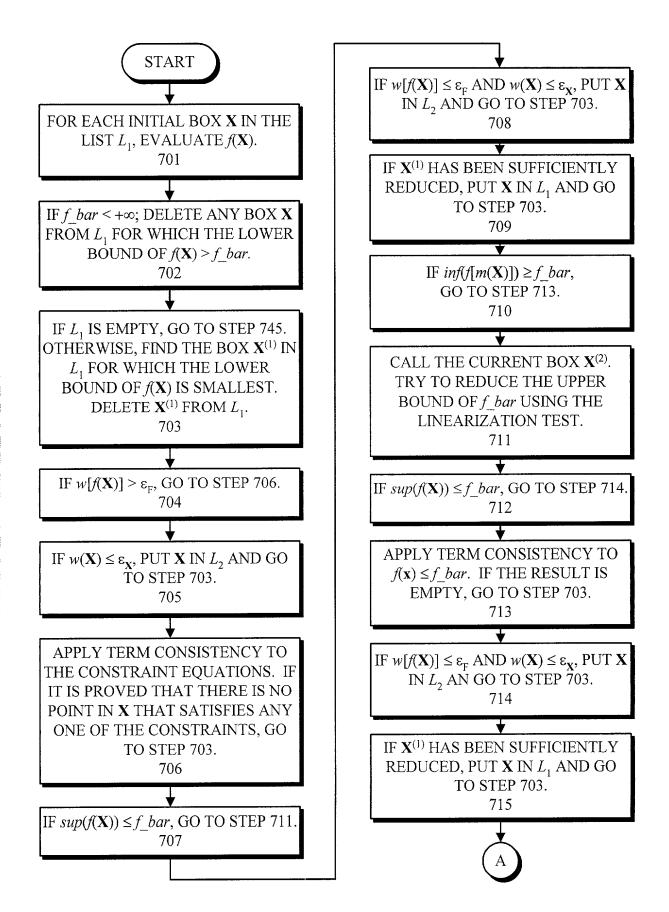
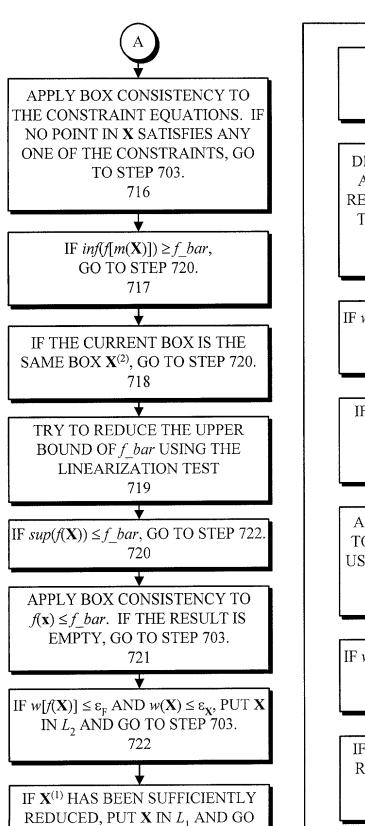


FIG. 7A



TO STEP 703.

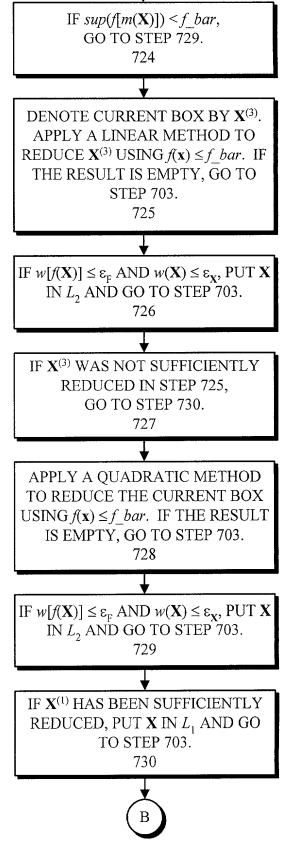


FIG. 7B



IF THE LINEARIZAATION TEST FOR THE CONSTRAINTS IS SATISFIED, SOLVE THE LINEARIZED CONSTRAINTS USING THE INTERVAL NEWTON METHOD. OTHERWISE, GO TO STEP 744.

731

REPLACE n-r OF THE VARIABLES BY THEIR INTERVAL BOUNDS. RENAME THE REMAINING VARIABLES AS  $x_p$ , ... $x_r$ . THEN LINEARIZE THE CONSTRAINT FUNCTIONS AS FUNCTIONS OF THE VARIABLES  $x_p$  ... $x_r$ . COMPUTE AN APPROXIMATE INVERSE **B** OF THE APPROXIMATE CENTER OF THE JACOBIAN  $\mathbf{J}(\mathbf{x}, \mathbf{X})$ .

732

PRECONDITION THE LINEARIZED SYSTEM. IF THE PRECONDITIONED COEFFICIENT MATRIX IS REGULAR, FIND THE HULL OF THE LINEARIZED SYSTEM. IF THE RESULT IS EMPTY, GO TO STEP 703.

733

IF  $w[f(\mathbf{X})] \le \varepsilon_{\mathrm{F}}$  AND  $w(\mathbf{X}) \le \varepsilon_{\mathbf{X}}$ , PUT  $\mathbf{X}$  IN  $L_2$  AND GO TO STEP 703.

ANALYTICALLY MULTIPLY THE NONLINEAR SYSTEM OF CONSTRAINT EQUATIONS BY THE MATRIX **B**. DO SO WITHOUT REPLACING ANY VARIABLES BY THEIR INTERVAL BOUNDS. REPLACE THE FIXED VARIABLES BY THEIR INTERVAL BOUNDS. APPLY TERM CONSISTENCY TO SOLVE THE  $i^{th}$  VARIABLE FOR i=1,...,r. IF THE RESULT IS EMPTY, GO TO STEP 703.

735

IF  $w[f(\mathbf{X})] \le \varepsilon_F$  AND  $w(\mathbf{X}) \le \varepsilon_X$ , PUT  $\mathbf{X}$  IN  $L_2$  AND GO TO STEP 703.



FIG. 7C



APPLY TERM CONSISTENCY TO SOLVE THE  $i^{th}$  NONLINEAR EQUATION OF THE PRECONDITIONED NONLINEAR SYSTEM FOR THE  $i^{th}$  VARIABLE FOR i=1,...,r. IF THE RESULT IS EMPTY, GO TO STEP 703. IF THE EXISTENCE OF A FEASIBLE POINT IS PROVED, USE THE RESULT TO UPDATE  $f\_bar$ .

737

IF  $w[f(\mathbf{X})] \le \varepsilon_F$  AND  $w(\mathbf{X}) \le \varepsilon_{\mathbf{X}}$ , PUT  $\mathbf{X}$  IN  $L_2$  AND GO TO STEP 703.

APPLY BOX CONSISTENCY TO SOLVE FOR THE  $i^{th}$  NONLINEAR EQUATION OF THE PRECONDITIONED NONLINEAR SYSTEM FOR THE  $i^{th}$  VARIABLE FOR i=1,...,r. IF THE RESULT IS EMPTY, GO TO STEP 703.

739

IF  $w[f(\mathbf{X})] \le \varepsilon_F$  AND  $w(\mathbf{X}) \le \varepsilon_{\mathbf{X}}$ , PUT  $\mathbf{X}$  IN  $L_2$  AND GO TO STEP 703.

IF THE REGULARITY TEST FOR THE JOHN CONDITIONS IS SATISFIED, SOLVE THE JOHN CONDITIONS (IN STEP 742) USING THE INTERVAL NEWTON METHOD. IF NOT, GO TO STEP 744.

741

APPLY ONE STEP OF THE INTERVAL NEWTON METHOD FOR SOLVING THE JOHN CONDITIONS. IF THE RESULT IS EMPTY, GO TO STEP 703. IF THE EXISTENCE OF A SOLUTION OF THE JOHN CONDITIONS IS PROVED, UPDATE  $f\_bar$ .

742

IF THE BOX  $\mathbf{X}^{(1)}$  HAS BEEN SUFFICIENTLY REDUCED, PUT  $\mathbf{X}$  IN  $L_1$  AND GO TO STEP 703.

743



FIG. 7D



ANY PREVIOUS STEP THAT USED TERM CONSISTENCY, A NEWTON STEP, OR A GAUSS-SEIDEL STEP MIGHT HAVE GENERATED GAPS IN THE INTERVAL COMPONENTS OF  ${\bf X}$ . MERGE ANY OVERLAPPING GAPS. SPLIT THE BOX. PLACE THE SUBBOXES GENERATED BY SPLITTING IN  $L_1$  AND GO TO STEP 703.

744

IF  $L_2$  IS EMPTY, THERE IS NO FEASIBLE POINT IN  $\mathbf{X}^{(0)}.\,$  GO TO STEP 750. 745

IF  $f\_bar < \infty$  AND THERE IS ONLY ONE BOX IN  $L_2$ , GO TO STEP 750. 746

FOR EACH BOX **X** IN  $L_2$ , IF  $sup(f[m(\mathbf{X})]) < f\_bar$ , TRY TO PROVE EXISTENCE OF A FEASIBLE POINT. USE THE RESULTS TO UPDATE  $f\_bar$ .

747

DELETE ANY BOX **X** FROM  $L_2$  FOR WHICH LOWER BOUND OF  $f(\mathbf{X}) > f\_bar$ . 748

DENOTE REMAINING BOXES  $X^{(1)}$ , ...,  $X^{(s)}$  in  $L_2$ . DETERMINE

$$\underline{F} = \min_{1 \le i \le s} \underline{f}(\mathbf{X}^{(i)}) \text{ AND } \overline{F} = \max_{1 \le i \le s} f(\mathbf{X}^{(i)}).$$

749

TERMINATE. 750

FIG. 7E